gegebenen Versuchsbedingungen $(a\,|\,\Delta E\,|)/(h\,v) > 1$ ist.

Bei Erhöhung der Stoßzahl in der Ionenquelle und Steigerung der Primärionenintensität stellt die Kombination aus Vierpol-Massenspektrometer und UV-Monochromator ein gutes Instrument dar zur Untersuchung von Reaktionen angeregter Moleküle im neutralen und Ionenzustand mit Gasen.

Die hier vorliegenden Messungen sind vorläufige Ergebnisse einer vor dem Abschluß stehenden Untersuchung, über die später ausführlich berichtet werden soll.

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On the 'Stability' of the Plasma Boundary Layer

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If a uniform plasma of particle densities $(n_\pm)$ and temperatures $(T_\pm)$ is in contact with an insulated wall it develops a disturbed region near the wall which is called the boundary layer. In this boundary layer we distinguish several model regions [1], one of them being the inertia limited zone (ILZ) immediately in front of the wall. Its extension is of the order of one ion mean free path.

It is important to know whether the boundary layer has a stationary solution, or better, whether it is stable.

BOHM [2] has analysed a part of this problem by investigating whether the space charge region (SCR) of the boundary layer introduces a non-stationarity or not. His well-known 'stability criterion' claims that the space charge region is 'stable' only if the kinetic energy of the ions entering the SCR is larger or equal to $k\,T_-/2$. His argument is based on the following assumptions:

1. plane geometry,
2. BOLTZMANN distribution of the electrons,
3. free fall of the ions,
4. negligible electric field (E_0) at the edge of the SCR,
5. equal particle densities $(n_{-0} = n_{+0})$ at the edge of the SCR.

The balic assumptions (2, 3) for the particle motion are actually characteristic for the inertia limited region. The extension of the space charge region may be larger or smaller than that of this inertia limited zone.

If it is larger, then BOHM's calculations should be restricted to the ILZ. If it is smaller, then his calculations should be extended to the whole ILZ since the part of the inertia limited zone outside the SCR might well have a bearing on the question of stationarity and stability.

Consequently we investigate here whether there is a stationary solution for the inertia limited region and whether its existence is limited by BOHM's criterion.

In doing this we take into account that at the edge of the ILZ, assumptions (4) and (5) do not necessarily hold. As a matter of fact from the calculation of the diffusion disturbed region [1] we can estimate the values of $\delta = n_{-0}/n_{+0}$ and E_0 and find that they may violate the conditions (4) and (5). The importance of the non-vanishing field at the edge of the SCR has already been mentioned by HALL [3].

Under these circumstances the stationary state of the inertia limited region is described by

$$\frac{d^2\eta}{ds^2} = \left(1 + \frac{2}{\gamma}\,\eta\right)^{-\frac{1}{2}} - \delta \exp(-\eta) \tag{1}$$

where we have used

$$\eta = eV/(k\,T_-), \quad s = x/l_D, \quad \gamma = M\,v^2_{+0}/(k\,T_-),$$
$$\delta = n_{-0}/n_{+0}, \quad l_D^2 = k\,T_-/(4\,\pi\,e^2\,n_{+0}). \tag{2}$$

V ist the negative of the electrostatic potential and v_{+0} is the velocity of the ions with mass M entering the ILZ.

Equation (1) may be integrated once, yielding

$$\left(\frac{d\eta}{ds}\right)^2 = 2\,\gamma\left[\left(1 + \frac{2}{\gamma}\,\eta\right)^{\frac{1}{2}} - 1\right] + 2\,\delta[\exp(-\eta) - 1] + \varepsilon_0^2 \tag{3}$$

where we have used the boundary conditions

$$\eta_0 = 0, \quad (d\eta/ds)_0 = \varepsilon_0. \tag{4}$$

Because of the positive definite character of the left hand side of equation (3), BOHM's criterion

$$\gamma \geqq 1 \tag{5}$$

would follow from an expansion of the right hand side of equation (3) if we made use of the assumptions (4) and (5).

However, according to the above argument these assumptions may not be true at the edge of the ILZ and consequently the situation may be different.

To show this we find for a given value of ε_0 and δ the lower limit of γ for which the right hand side of equation (3) is positive definite for all values of η.

This has been done by choosing a value of ε_0 and then finding the value of $\eta = \eta^*$ for which the right hand side of equation (3) has a minimum. One obtains

$$\delta = \left(1 + \frac{2}{\gamma}\,\eta^*\right)^{-\frac{1}{2}} \cdot \exp(\eta^*). \tag{6}$$

1 UCRL Report No. 10128.
2 D. BOHM, in the 'Characteristics of Electrical Discharges in Magnetic Fields', A. GUTHRIE and R. K. WAKERLING, eds., McGraw-Hill Book Co., Inc., New York 1949, Chap. III.
3 L. S. HALL, Phys. Fluids 4, 388 [1961].

For this value η^* the right hand side of equation (3) must be at least zero. This defines a critical maximum value δ_c for a given set of ε_0 and γ by

$$2\gamma\left[\left(1+\frac{2}{\gamma}\eta^*\right)^{\frac{1}{2}}-1\right]+2\,\delta_c[\exp(-\eta^*)-1]+\varepsilon_0{}^2=0\,. \quad (7)$$

The elimination of η^* from equation (6) and (7) gives for a fixed parameter ε_0, a relation between δ_c and γ as demonstrated in the figure.

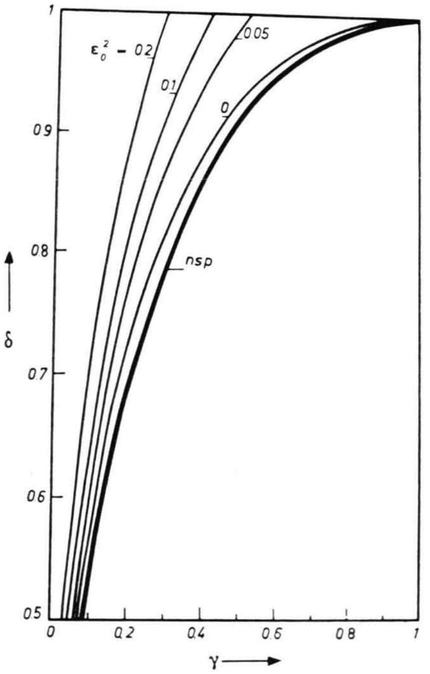

Fig. 1. The effects of the boundary values (δ, ε_0, γ) on the stationarity criterion for the inertia limited zone.

The figure also contains an additional curve labeled (nsp). This curve separates the parameter ranges in which a partial negative space charge occurs from those where only positive space charge is present. The corresponding relation is easily found by requiring that the space charge be zero at its minimum value

$$\delta_{sp}=\gamma^{\frac{1}{2}}\exp\{(1-\gamma)/2\}. \quad (8)$$

This latter distinction seems essential since it has been argued[4] that configurations with a partial negative space charge should be excluded.

The results shown in the figure may be summarized as follows:

According to Bohm's criterion, no stationary solution should exist within the whole range of this figure. We see, however, that stationary solutions do exist for $\gamma<1$ if variations in the density ratio ($\delta<1$) and a finite field ($\varepsilon_0\neq0$) at the edge of the ILZ are taken into account. For a given value of ε_0 all combinations of δ and γ to the right and below the curve labeled ε_0 yield stationary solutions. If the chosen combination lies between the ε_0 curve and the (nsp) curve, we have a partial negative space charge. If the chosen combination lies to the right and below the (nsp) curve then we have positive space charge only.

From the preceeding we conclude that Bohm's criterion it not a necessary limitation for the boundary layer.

We further wish to investigate whether the criterion has sufficient character to ensure a stationary solution of the boundary layer.

The extension of the inertia limited region is a fixed quantity. We can therefore integrate equation (3) over this region, obtaining a value for the wall potential (V_W). This value of V_W depends on the parameters which enter into equation (3) and these in turn depend on the plasma properties n_{+0}, n_{-0}, kT_-, E_0 at the edge of the inertia limited region. Consequently we get the wall potential as a function of these parameters in the general form

$$V_W=V_W(n_{+0},\,n_{-0},\,k\,T_-,\,E_0)\,. \quad (9)$$

Since we are dealing with an insulated wall, the total electric wall current in the stationary state has to be zero. This current depends on the value of the wall potential and the plasma parameters n_{+0}, n_{-0}, kT_-, E_0 in the general form

$$J_T(n_{+0},\,n_{-0},\,k\,T_-,\,E_0,\,V_W)=0\,. \quad (10)$$

The elimination of V_W from (9) and (10) yields the condition

$$G(n_{+0},\,n_{-0},\,k\,T_-,\,E_0)=0\,. \quad (11)$$

In general the plasma parameters at the edge of the inertia limited zone will not fulfil this condition. As a matter of fact they are prescribed by the values at the edge of the diffusion disturbed region — if we decide to join these two regions.

Consequently we should conclude that the ILZ formed when a plasma is bounded by an insulated wall can in general not have a stationary solution. This is true even if Bohm's criterion is fulfilled.

Therefore his criterion seems not sufficient.

We do not overlook the existence of a transition region between the diffusion disturbed region and the inertia limited region which could possibly adjust the parameter values to avoid the non-stationarity. We are in the process of investigating this problem and can exclude such a possibility at least for a wide range of densities and temperatures.

One more remark about the commonly used term 'stability' in this connection. The condition that the right hand side of equation (3) be always positive assures the existence of a meaningful solution of the stationary equation. It does not 'per se' ensure a stable solution. This condition, therefore, produces a 'stationarity criterion' and not a 'stability criterion'. The stability of the stationary state is another question which cannot be answered solely on the basis of equation (3).

[4] G. J. Schultz and S. C. Brown, Phys. Rev. **98**, 1642 [1955].